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AMPLITUDE–FREQUENCY RESPONSE OF AN ALUMINIUM CANTILEVER BEAM DETERMINED WITH PIEZOELECTRIC TRANSDUCERS

AMPLITUDNO-FREKVENČNI ODZIV KONZOLNEGA NOSILCA IZ ALUMINIJA, UGOTOVLJEN S PIEZOELEKTRIČNIMI PRETVORNIKI

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This work is focused on the creation of an appropriate finite-element model of an aluminum cantilever beam using a pair of piezoelectric patch transducers. Thanks to the reversible behavior of the piezoelectric effect each patch transducer can represent either an actuator or a sensor. For a precise prediction of the amplitude values in the numerical simulations each transducer is calibrated before being attached to the beam with strain gauges. From these experiments piezoelectric properties of each piezoelectric patch are obtained. The cantilever beam is actuated with a voltage signal applied to one of the patches. The signal is a linear chirp (sine wave with a swept frequency) with a sufficient range to affect the selected natural frequencies. The time response of the beam from the piezoelectric sensor and, alternatively, from the laser position sensor is transformed with the STFT algorithm to obtain the characteristics of the time-frequency domain (spectrogram). The finite-element model of the cantilever beam with the piezoelectric patches was created using 3D solid structural and piezoelectric bricks in Ansys. The time response of the model to the chirp voltage signal was determined with a transient analysis. The amplitude/frequency characteristics are compared with the experimental results.

Keywords: piezoelectric materials, frequency spectrum, finite-element analysis

To delo obravnava izdelavo primernega modela z metodo končnih elementov konzolnega nosilca iz aluminija z uporabo para piezoelektričnih pretvornikov v obliki obliža. Zaradi reverzibilnega vedenja piezoelektričnega pojava je lahko vsak obližast pretvornik aktuator ali senzor. Za natančno napovedovanje vrednosti amplitude pri numeričnih simulacijah je bil vsak pretvornik pred namestitvijo na nosilec kalibriran z napetostnimi lističi. Iz teh preizkusov so dobljene piezoelektrične lastnosti vsakega piezoelektričnega obliža. Konzolni nosilec je bil aktiviran s signalom električne napetosti, uporabljene na enem od obližev. Signal je linearno cvrčanje (sinus s šablonirano frekvenco) s primernim območjem, da se vpliva na izbrane naravne frekvence. Časovni odziv nosilca iz piezoelektričnega senzorja in alternativno s položaja laserskega senzorja je pretvorjen s STFT-algoritmom, da se dobi značilnosti vedenja čas – frekvenca (spektrogram). Izdelan je bil model s končnimi elementi konzolnega nosilca s piezoelektričnimi obliži. Z uporabo 3D strukturnih in piezoelektričnih opek v Ansys in z uporabo končnih elementov je bil izdelan model konzolnega nosilca s piezoelektričnimi obliži. Časovni odziv modela na cvrčeč signal napetosti je bil določen s prehodno analizo. Značilnosti amplitude in frekvence so primerjane z eksperimentalnimi rezultati.

Ključne besede: piezoelektrični material, spekter frekvenc, analiza končnih elementov

1 INTRODUCTION

Automatic detection of impacts and hidden defects in structures (denoted as structural health monitoring, SHM) is the present-day trend in the non-destructive testing. The data obtained from the sensors applied to a structure are transmitted to the control system, which evaluates the state of the structure and responses appropriately (e.g, it enables the warning system). Especially the structures made of composite materials affected by hidden defects such as fibre debonding or delamination[1] call for an integration of the novel SHM methods.

Two approaches are usually distinguished in SHM: the passive and active ones. In a passive system "natural" impulses like impacts or crack propagation create stress waves that are sensed by a grid of sensors. The source can be then localized and reconstructed.[2] In an active SHM system the health of a structure is assessed by evaluating its response to specific actuating signals. One way of establishing damage is by detecting the changes in a structure's modal properties (particularly the basic natural frequency),[3] providing the global information on its state. Another way, denoted as the pitch-catch technique, uses the scattering of stress waves when the actuating signal approaches a structural defect.[4]

The sensors and actuators in SHM systems are made of smart materials that have a capability to convert various kinds of energy, e.g., piezoelectric materials (Rochelle salt, tourmaline or artificially produced ceramics) respond to a mechanical deformation by generating electric voltage and vice versa. Therefore, piezoelectric materials can be used as both sensors and actuators.

This work is focused on creating a reliable FE model of a piezoelectric transducer used in SHM. The model is then tested in the case when two patches are applied to a cantilever beam. An appropriate FE model of a piezoelectric patch is the key part for designing a future SHM system.

2 MODEL OF THE PIEZOELECTRIC MATERIAL

The piezoelectric material can be described with a set of constitutive equations:

$$\begin{bmatrix}\sigma\\ D\end{bmatrix}=\begin{bmatrix}C & -e^{\mathrm{T}}\\ e & \mu\end{bmatrix}\cdot\begin{bmatrix}\varepsilon\\ E\end{bmatrix} \tag{1}$$

where σ [Pa] is stress vector 6 × 1, C [Pa] is the matrix of elastic coefficients 6 × 6, ε is strain vector 6 × 1, D [C/m^2] is electric displacement vector 3 × 1, E [V/m] is electric-field intensity vector 3 × 1, μ is dielectric matrix 3 × 3 (with electric permittivity constants on its diagonal) and e [C/m^2] is piezoelectric stress matrix 3 × 6:

$$e=\begin{bmatrix}0 & 0 & 0 & 0 & e_{15} & 0\\ 0 & 0 & 0 & e_{24} & 0 & 0\\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0\end{bmatrix} \tag{2}$$

The rows of matrix e denote the direction of the electric field and the columns refer to the strain components, e.g., constant e_{32} of the piezoelectric actuator quantifies strain ε_2 induced by the electric field in transversal direction 3.

In the producer's datasheet piezoelectric strain matrix d is listed instead of e. These matrices are related using the stiffness matrix:

$$e^{\mathrm{T}}=Cd^{\mathrm{T}} \tag{3}$$

A transversally isotropic material model is considered for the piezoelectric ceramic with the main direction of anisotropy identical with the direction of polarity. Stiffness matrix C is obtained with the inversion of compliance matrix S:

$$S=\begin{bmatrix}\frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0\\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0\\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}}\end{bmatrix} \tag{4}$$

where E_1 is the Young's modulus in the direction perpendicular to the plane of isotropy, while $E_2 = E_3$ are the Young's moduli in the plane of isotropy. $G_{12} = G_{13}$ are the shear moduli in the planes perpendicular to the plane of isotropy, $G_{23} = G_{32}$ are the shear moduli in the plane of isotropy and are defined with:

$$G_{23}=\frac{E_2}{2(1+\nu_{23})}=\frac{E_3}{2(1+\nu_{32})} \tag{5}$$

Poisson's ratios $\nu_{12} = \nu_{13}$ give a measure of the extension (compression) in the plane of isotropy due to the extension (compression) in the direction perpendicular to this plane and vice versa. $\nu_{23} = \nu_{32}$ are the Poisson's ratios in the plane of isotropy.

3 DETERMINATION OF PIEZOELECTRIC COEFFICIENTS OF THE PATCHES

The piezoelectric patches used in the experiments are DuraAct P-876.A12 with a layer of piezoelectric ceramic (type PIC-255 [5]) with the dimensions of 50 mm × 30 mm × 0.2 mm. The top and bottom surfaces of the ceramic are silvered and connected to the soldering pads. Due to the high fragility the ceramic is embedded in a protective polymeric foil with the dimensions of 61 mm × 35 mm × 0.5 mm. The mechanical and electrical properties of the constituent materials are presented in **Tables 1** and **2**. Material properties of the piezoelectric ceramic were determined by the producer.[5]

Table 1: Material properties
Tabela 1: Lastnosti materiala

		Units	PZT	Foil	Al
Young's modulus	E	[GPa]	See Tab.2	3	68
Poisson's ratio	ν	[–]		0.3	0.3
Density	ρ	[kg/m^3]	7800	1580	2777
Relative electric permittivity	μ_1/μ_0*	[–]	1650	–	–
	μ_2/μ_0	[–]	1650		
	μ_3/μ_0	[–]	1750		
Piezoelectric coefficients	e_{31},e_{32}	[C/m]	6.4	–	–
	e_{33}	[C/m]	–20.5	–	–
* μ_0 = 8.85418 × 10^{-12} F/m is vacuum permittivity					

Table 2: Elastic parameters of piezoelectric ceramic PIC-255
Tabela 2: Parametri elastičnosti piezoelektrične keramike PIC-255

E_1	[GPa]	62.1
$E_2 = E_3$	[GPa]	48.3
$\nu_{12} = \nu_{13}$	[–]	0.34
$\nu_{23} = \nu_{32}$	[–]	0.34
$G_{12} = G_{13}$	[GPa]	23.1
$G_{23} = G_{32}$	[GPa]	17.9

Although geometric and material properties of the piezoelectric patches are supposed to be similar for one type and production set, various results with differences up to 20 % were obtained using two patches of the same type and set. For this reason the piezoelectric properties of the patches used in following experiment need to be determined.

In the first experiment two pairs of strain gauges (HBM rosettes 6/650 RY91) were glued on the two

Figure 1: Piezoelectric patch with the applied strain gauges, one on the front side and one on the back side
Slika 1: Piezoelektrični obliž z napetostnimi lističi: eden spredaj in eden zadaj

piezoelectric patches (**Figure 1**), one on each side of a patch, to eliminate the influence of the minor patch curvature. The strain response to the applied static-electric voltage was measured and these values are presented in **Table 3**. The difference between these two patches loaded with 100 V was 14.4 %.

Table 3: Measured strains of the two patches (loaded with 100 V)
Tabela 3: Izmerjene napetosti dveh obližev (obremenjenih s 100 V)

Patch	1 / sensor	2 / actuator
ε_{11} by SG 1	10.27×10^{-5}	11.74×10^{-5}
ε_{11} by SG 2	9.38×10^{-5}	11.23×10^{-5}
ε_{11} – averaged	9.83×10^{-5}	11.49×10^{-5}

The piezoelectric-matrix coefficients were identified using a FE model in Ansys v.14. The model was created using 3D hexagonal elements: 20-node piezoelectric bricks (SOLID 226) for PZT and 20-node structural bricks (SOLID 186) for the protective foil. The piezoelectric elements have an additional degree of freedom for the electric potential in each node. One layer of these elements was used, which was sufficient for an approximation of the electric potential across the thickness of a patch. The nodes of the piezoelectric elements in the top and bottom surfaces are represented by the silver electrodes, where the relevant electric potential is applied.

The model parameter e_{31} was optimized to match the experimental data. Coefficients e_{32} and e_{33} were calculated as $e_{32} = e_{31}$ and e_{33} was chosen to maintain the mutual ratio of $e_{33}/e_{31} \approx -2.5$. The resulting values are presented in **Table 4**.

Table 4: Identified piezoelectric coefficients of the two patches
Tabela 4: Ugotovljeni piezoelektrični koeficienti pri dveh obližih

Patch	1 / sensor	2 / actuator
e_{31}, e_{32} [C/m]	7.1	8.3
e_{33} [C/m]	–17.8	–20.8

4 FREQUENCY RESPONSE OF THE CANTILEVER BEAM

The calibrated patches were glued to an aluminium beam with the dimensions of 1000 mm × 30 mm × 3 mm, each on one side. The beam was clamped at 100 mm of its length and 10 mm from the patches.

The patches were connected to the National Instruments data acquisition system (NI CompactDAQ) supplied with an actuating module NI 9215, sensing module NI 9236 and an amplifier. The beam was actuated by one of the patches and the vibration was sensed by another patch and laser position sensor OptoNCDT. The experimental set-up is presented in **Figure 2**.

The actuating signal was a linear chirp (sine wave with a linearly swept frequency) defined with the following equation:

$$x(t)=X\cdot\sin\left[\varphi_0+2\pi\left(f_0+\frac{k}{2}t^2\right)\right] \tag{6}$$

where X is the amplitude, φ_0 is the initial phase, f_0 is the initial frequency, k is the chirp rate defined by:

$$k=\frac{f_1-f_0}{t_1} \tag{7}$$

where f_1 denotes the final frequency and t_1 is the final time.

Table 5: Parameters of the actuating signal
Tabela 5: Parametri vzbujevalnega signala

Amplitude	X	[V]	75
Initial phase	φ_0	[rad]	0
Initial frequency	f_0	[Hz]	0
Final frequency	f_1	[Hz]	100
Final time	t_1	[s]	20
Sampling frequency	f_s	[Hz]	50000

The properties of the actuating chirp signal are presented in **Table 5**. The range of frequencies was chosen to contain the lowest natural frequencies of the beam for the relevant two out-of-plane bending modes.

The time-voltage responses of the piezoelectric sensor and laser sensor were recorded (**Figure 3**). A short-time Fourier transform (STFT) was performed to obtain a spectrogram (**Figure 4**). The length of the Hanning window in STFT was set to 10^5 samples providing a frequency precision of 0.5 Hz. The two lowest natural

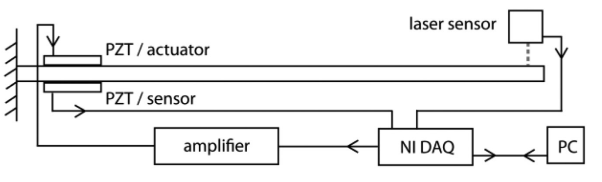

Figure 2: Experimental set-up
Slika 2: Eksperimentalni sestav

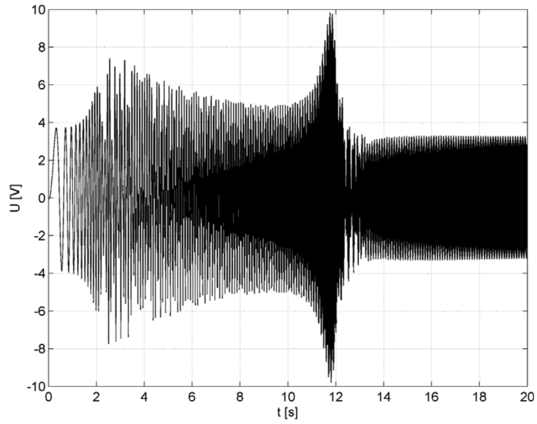

Figure 3: Time response of the cantilever beam measured with PZT
Slika 3: Časovni odziv konzolnega nosilca, izmerjen s PZT

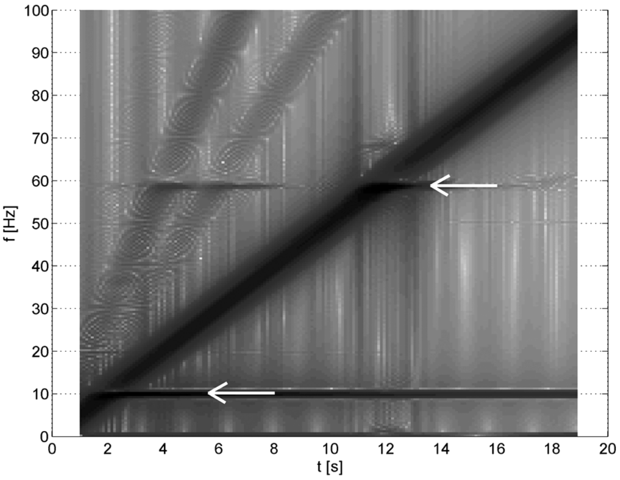

Figure 4: Spectrogram of the cantilever beam, the two lowest natural frequencies are marked with arrows
Slika 4: Spektrogram konzolnega nosilca; puščici prikazujeta dve najnižji naravni frekvenci

frequencies can be detected in the spectrogram. In **Table 6** the experimental results are compared with the numerically calculated natural frequencies.

Table 6: Comparison of natural frequencies
Tabela 6: Primerjava naravnih frekvenc

	FEM [Hz]	Experiment [Hz]
1st	9.9	10.0
2nd	60.5	60.0

5 AMPLITUDE RESPONSE OF THE CANTILEVER BEAM TO THE HARMONIC LOADING

To obtain the amplitude response the structure was loaded with a harmonic sine wave with a low frequency (1 Hz) with a duration of 40 s to approach steady oscillations. The amplitude was 100 V and the sampling frequency was 25 kHz. The deflection of the beam tip was measured with the laser sensor and compared with the results of the FE static analysis (**Table 7**). The difference between the experiment and FE was 2.4 %.

Table 7: Comparison of the experimental and numerical results
Tabela 7: Primerjava eksperimentalnih in numeričnih rezultatov

	Beam-tip deflection u_z
Experiment	0.206 mm
FEA	0.201 mm
Difference	2.4 %

6 CONCLUSION

A numerical model of a piezoelectric transducer was created in Ansys using three-dimensional piezoelectric and structural finite elements. The piezoelectric coefficients of each patch were calibrated using strain gauges and it was found that they differ by 14 %.

The FE model of the piezoelectric transducer was tested on a problem of bending an aluminum cantilever beam. The two lowest natural frequencies were determined experimentally and compared with the results of the FE modal analysis with sufficient match for the given frequency precision.

The amplitude of deflection of the beam loaded with the low-frequency voltage was measured with the laser sensor and compared to the result of the FE static analysis with a difference of 2.4 %.

This numerical model proved to be suitable for designing the SHM systems based on the change in the structure's natural frequencies. The reliability of the model will be further tested for the case of transient problems such as those used in the pitch-catch SHM systems.

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