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A GENERALIZED THEORY OF PLASTICITY

POSPLOŠENA TEORIJA PLASTIČNOSTI

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A closed solution of the plane problem of the generalized theory of plasticity and a model of the complex plastic medium were theoretically developed. Solutions with the use of the deformation theory and the theory of plastic yielding were developed. The solution for a simple strengthening medium was deduced.

Key words: metal plasticity, analytical solution, mathematical model, plastic medium, process parameters

Razvita sta bila zaprta rešitev splošne teorije plastičnosti in teoretičen model kompleksnega plastičnega medija. Opredeljene so bile rešitve z uporabo teorije deformacije in teorije plastičnega tečenja. Razvita je bila rešitev za preprost utrditveni medij.

Ključne besede: plastičnost kovin, analitična rešitev, matermatični model, plastični medij, parametri procesa

1 INTRODUCTION

A characteristic of the new method based on a closed solution of the plane problem of theory of plasticity is a simplified analysis of the deformation mode of the medium and the theoretical connection to the medium mechanical characteristics through the process parameters. The analytical solution of the plane problem of the theory of plasticity for a strengthening medium is known.[1] The developed complex model for the strengthening of the plastic medium is based on the shear resistance to the plastic deformation and is a function of the coordinates of the nucleus of deformation. This approach offers a new possibility to evolve a new solution for a problem, including the generalized theory of plasticity. The approach includes equations and criteria: an equilibrium equation, and the criteria of yielding, the equation of incompressibility, of the deformation rate and the deformation as well as equations of continuity of the deformation rate and the deformation:

– the equilibrium equations

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}=0;\ \frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_y}{\partial y}=0$$

– the criterion of yielding

$$(\sigma_x-\sigma_y)^2+4\cdot\tau_{xy}^2=4\cdot k^2$$

– the constraint equations for the rates of deformation and deformation

$$\frac{\sigma_x-\sigma_y}{2\cdot\tau_{xy}}=\frac{\xi_x-\xi_y}{\gamma'_{xy}}=F_1;\ \frac{\sigma_x-\sigma_y}{2\cdot\tau_{xy}}=\frac{\varepsilon_x-\varepsilon_y}{\gamma_{xy}}=F_2 \quad (1)$$

– the equations of incompressibility for the rates of deformation and the deformation

$$\xi_x+\xi_y=0;\ \varepsilon_x+\varepsilon_y=0$$

– the equation of continuity for the deformation rates and the deformation

$$\frac{\partial^2\xi_x}{\partial y^2}+\frac{\partial^2\xi_y}{\partial x^2}=\frac{\partial^2\gamma'_{xy}}{\partial y\partial x};$$

$$\frac{\partial^2\varepsilon_x}{\partial y^2}+\frac{\partial^2\varepsilon_y}{\partial x^2}=\frac{\partial^2\gamma_{xy}}{\partial y\partial x}$$

– the equation of heat conductivity

$$a^2\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=\frac{\partial T}{\partial t}$$

The model of the complex plastic medium is defined with

$$T_{\mathrm{i}}=\chi\cdot(H_{\mathrm{i}})^{m_1}\cdot(\Gamma_{\mathrm{i}})^{m_2}\cdot(T)^{m_3} \quad (2)$$

The system of equations (1) includes the equations of the deformation theory of plasticity and the theory of plastic yielding with the addition of the equation of heat conductivity.[2] The model (2) is a real strengthening medium with the boundary conditions for stresses[3]

$$\tau_{\mathrm{n}}=T_{\mathrm{i}}\cdot\sin[A\Phi-2\alpha],\ T_{\mathrm{i}}=k$$

or

$$\tau_{\mathrm{n}}=\frac{\sigma_x-\sigma_y}{2}\cdot\sin 2\alpha-\tau_{xy}\cdot\cos 2\alpha \quad (3)$$

The additional conditions are given by the specific contact forces (3) of the change of friction according to the sinusoidal law of deformational and high-speed strain hardening. All the intensities and the temperature depend on the coordinates of the deformation nucleus.

2 THEORETICAL DEVELOPMENT

With the aim to obtain the model (2), let us consider three second-order equations in form of non-uniform hyperbolic partial derivations:

$$\frac{\partial^2 \tau_{xy}}{\partial x^2}-\frac{\partial^2 \tau_{xy}}{\partial y^2}=2\cdot\frac{\partial^2}{\partial y\partial x}\sqrt{k^2-\tau_{xy}^2},$$
$$\frac{\partial^2 \xi_x}{\partial y^2}-\frac{\partial^2 \xi_x}{\partial x^2}=2\cdot\frac{\partial^2}{\partial y\partial x}\frac{1}{F_1}\cdot\xi_x \qquad (4)$$
$$\frac{\partial^2 \varepsilon_x}{\partial y^2}-\frac{\partial^2 \varepsilon_x}{\partial x^2}=2\cdot\frac{\partial^2}{\partial y\partial x}\frac{1}{F_2}\cdot\varepsilon_x$$

The boundary conditions (3) correspond to the substitution $\tau_{xy}=k\cdot\sin A\Phi$. A complex dependence of the coordinates is assumed with $k=f(\Gamma_i,H_i,T,x,y)$. In this case, $k=C_\sigma\cdot\exp\theta'$, with $\theta'=f(\Gamma_i,H_i,T,x,y)$, with Γ_i, H_i, T standing for the intensity of the deformation, the rates of deformation and the temperature.

The derivatives are taken as for the complex function,[4] and after substitution in the first equation (3) we obtain:

$$\Big\{(\theta'_H\cdot H_x+\theta'_s\cdot\Gamma_x+\theta'_t\cdot T_x)_x+\big[(\theta'_H\cdot H_x+\theta'_s\cdot\Gamma_x+$$
$$+\theta'_t\cdot T_x)+A\Phi_y\big]^2-(\theta'_H\cdot H_y+\theta'_s\cdot\Gamma_y+\theta'_t\cdot T_y)_y-$$
$$-\big[(\theta'_H\cdot H_y+\theta'_s\cdot\Gamma_y+\theta'_t\cdot T_y)-A\Phi_x\big]^2+2A\Phi_{xy}\Big\}\cdot$$
$$\cdot\sin(A\Phi)+\Big\{2\cdot\big[(\theta'_H\cdot H_x+\theta'_s\cdot\Gamma_x+\theta'_t\cdot T_x)+A\Phi_y\big]\cdot$$
$$\cdot\big[A\Phi_x-(\theta'_H\cdot H_y+\theta'_s\cdot\Gamma_y+\theta'_t\cdot T_y)\big]+A\Phi_{xx}-$$
$$-A\Phi_{yy}-2\cdot(\theta'_{HH}\cdot H_x\cdot H_y+\theta'_H\cdot H_{xy}+$$
$$-\theta'_{ss}\cdot\Gamma_x\cdot\Gamma_y+\theta'_s\cdot\Gamma_{xy}+\theta'_{tt}\cdot T_x\cdot T_y+\theta'_t\cdot T_{xy})\Big\}\cdot$$
$$\cdot\cos(A\Phi)=0 \qquad (5)$$

Equation (5) is equal to zero if the parts in the square brackets are equal to zero. Then,

$$\theta'_H\cdot H_x+\theta'_s\cdot\Gamma_x+\theta'_t\cdot T_x=A\Phi_y$$
$$\theta'_H\cdot H_y+\theta'_s\cdot\Gamma_y+\theta'_t\cdot T_y=A\Phi_x$$
$$(\theta'_H\cdot H_x+\theta'_s\cdot\Gamma_x+\theta'_t\cdot T_x)_x=-A\Phi_{yx}$$
$$(\theta'_H\cdot H_y+\theta'_s\cdot\Gamma_y+\theta'_t\cdot T_y)_y=-A\Phi_{xy}$$
$$A\Phi_{yy}=-(\theta'_{HH}\cdot H_x\cdot H_y+\theta'_H\cdot H_{xy}+\theta'_{ss}\cdot\Gamma_x\cdot\Gamma_y+$$
$$+\theta'_s\cdot\Gamma_{xy}+\theta'_{tt}\cdot T_x\cdot T_y+\theta'_t\cdot T_{xy})$$
$$A\Phi_{xx}=(\theta'_{HH}\cdot H_x\cdot H_y+\theta'_H\cdot H_{xy}+\theta'_{ss}\cdot\Gamma_x\cdot\Gamma_y+$$
$$+\theta'_s\cdot\Gamma_{xy}+\theta'_{tt}\cdot T_x\cdot T_y+\theta'_t\cdot T_{xy})$$

The operations with the complex function allow us to determine the exponent index as the sum of three functions accounting for the effect of the deformation degree and the rate, and of the temperature:

$$\theta'=-A\theta=\theta'_1+\theta'_2+\theta'_3=-(A'_1\theta+A'_2\theta+A'_3\theta)$$

The shear resistance and the components of the tensor of the stresses are:

$$k=C_\sigma\cdot\exp(-A'_1\theta)\cdot\exp(-A'_2\theta)\cdot\exp(-A'_3\theta)\cdot\sin(A\Phi) \qquad (6)$$

$$\sigma_x=C_\sigma\cdot\exp(-A'_1\theta)\cdot\exp(-A'_2\theta)\cdot\exp(-A'_3\theta)\cdot\cos(A\Phi)+$$
$$+\sigma_0+f(y)+C$$
$$\sigma_y=-C_\sigma\cdot\exp(-A'_1\theta)\cdot\exp(-A'_2\theta)\cdot\exp(-A'_3\theta)\cdot\cos(A\Phi)+$$
$$+\sigma_0+f(x)+C$$

with $\theta'_x=(\theta'_1)_x+(\theta'_2)_x+(\theta'_3)_x=-A\Phi_y$

$$\theta'_y=(\theta'_1)_y+(\theta'_2)_y+(\theta'_3)_y=A\Phi_x$$

By substituting the stress values into the equation of constraint we obtain:

$$\frac{\sigma_x-\sigma_y}{2\tau_{xy}}=\mathrm{ctg}\,A\Phi;\ \frac{\xi_x-\xi_y}{\gamma'_{xy}}=\mathrm{ctg}\,B_1\Phi$$
$$\mathrm{ctg}\,A\Phi=\mathrm{ctg}\,B_1\Phi=F_1$$
$$\frac{\varepsilon_x-\varepsilon_y}{\gamma'_{xy}}=\mathrm{ctg}B_2\Phi$$
$$\mathrm{ctg}\,A\Phi=\mathrm{ctg}\,B_2\Phi=F_2$$

It is possible to establish the relation between the shears and the linear figures of the deformation rates and the deformations. Taking into account the equations of non-compressibility we obtain:

$$\gamma'_{xy}=2\cdot\frac{1}{F_1}\cdot\xi_x=2\cdot\xi_x\cdot\mathrm{tg}\,B_1\Phi$$
$$\gamma_{xy}=2\cdot\frac{1}{F_2}\cdot\varepsilon_x=2\cdot\varepsilon_x\cdot\mathrm{tg}\,B_2\Phi$$

In order to simplify, we define:

$$\xi_x=C_\xi\cdot\exp\theta''_1\cdot\cos B_1\Phi$$
$$\varepsilon_x=C_\varepsilon\cdot\exp\theta''_2\cdot\cos B_2\Phi$$

By substituting these relations into the equations of continuity of the deformation rate and the deformation (1) or (4), we obtain:

$$\big[-\theta''_{1xx}-(\theta''_{1x}+B_1\Phi_y)^2+\theta''_{1yy}+(\theta''_{1y}-B_1\Phi_x)\big]\cdot\sin B_1\Phi+$$
$$+\big[2(B_1\Phi_x-\theta''_{1y})\cdot(\theta''_{1x}+B_1\Phi_y)+(B_1\Phi_{xx}-B_1\Phi_{yy})\big]\cdot$$
$$\cdot\cos B_1\Phi=2\cdot B_1\Phi_{xy}\cdot\sin B_1\Phi+2\cdot\theta''_{1xy}\cdot\cos B_1\Phi \qquad (7)$$

as well as

$$\big[-\theta''_{2xx}-(\theta''_{2x}+B_2\Phi_y)^2+\theta''_{2yy}+(\theta''_{2y}-B_2\Phi_x)\big]\cdot\sin B_2\Phi+$$
$$+\big[2(B_2\Phi_x-\theta''_{2y})\cdot(\theta''_{2x}+B_2\Phi_y)+(B_2\Phi_{xx}-B_2\Phi_{yy})\big]\cdot$$
$$\cdot\cos B_2\Phi=2\cdot B_2\Phi_{xy}\cdot\sin B_2\Phi+2\cdot\theta''_{2xy}\cdot\cos B_2\Phi \qquad (8)$$

Brackets identical to (5) appear in equations (7) and (8). For

$$(\theta''_1)_x=-B_1\Phi_y\quad(\theta''_1)_y=B_1\Phi_x$$
$$(\theta''_2)_x=-B_2\Phi_y\quad(\theta''_2)_y=B_2\Phi_x$$

the equations are transformed into identities, with, $\theta''_1=-B_1\theta$, $\theta''_2=-B_2\theta$ as the indices of the exponents of the functions determining the fields of the deformation rate and the deformation, $B_1\Phi$ and $B_2\Phi$ are the trigonometric functions determining the fields of the deformation rate and the deformation.

The expressions for the deformation rate and the deformation are:

$$\begin{aligned}\xi_x &= -\xi_y = C_\xi \cdot \exp\theta_1^{''} \cdot \cos B_1\Phi = \\ &= C_\xi \cdot \exp(-B_1\theta)\cdot \cos B_1\Phi \end{aligned} \tag{9}$$

$$\gamma'_{xy} = C_\xi \cdot \exp\theta_1^{''} \cdot \sin B_1\Phi = C_\xi \cdot \exp(-B_1\theta)\cdot \sin B_1\Phi$$

$$H_i = 2C_\xi \cdot \exp\theta_1^{''} = 2C_\xi \cdot \exp(-B_1\theta)$$

$$\begin{aligned}\varepsilon_x &= -\varepsilon_y = C_\varepsilon \cdot \exp\theta_2^{''} \cdot \cos B_2\Phi = \\ &= C_\varepsilon \cdot \exp(-B_2\theta)\cdot \cos B_2\Phi \end{aligned} \tag{10}$$

$$\gamma_{xy} = C_\varepsilon \cdot \exp\theta_2^{''} \cdot \sin B_2\Phi = C_\varepsilon \cdot \exp(-B_2\theta)\cdot \sin B_2\Phi$$

$$\Gamma_i = 2C_\varepsilon \cdot \exp\theta_2^{''} = 2C_\varepsilon \cdot \exp(-B_2\theta)$$

with $(\theta_1^{''})_y = B_1\Phi_x \quad (\theta_1^{''})_x = -B_1\Phi_y$

$(\theta_2^{''})_y = B_2\Phi_x \quad (\theta_2^{''})_x = -B_2\Phi_y$

With a comparison of expressions (9), (10) and (7) we can confirm that all the expressions have functional dependencies on the coordinates θ and Φ (the indices of the exponents and the examples of the trigonometric functions).

It is of interest to obtain similar dependencies for the solution for the temperature fields that could allow us to solve this task theoretically. Let us consider a differential equation for the stationary temperature field

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

For this case we look for the solution in the form of

$$T = C_T \cdot \exp(\theta_3^{''})\cdot(\sin B_3\Phi + \cos B_3\Phi) \tag{11}$$

with: $(\theta_3^{''})_x = -B_3\Phi_y,\ (\theta_3^{''})_y = -B_3\Phi_x$

We will demonstrate that expression (11) is a solution of the Laplace equation. By substituting the derivatives (11) into the equation of the heat conductivity and a simplification we obtain:

$$\begin{aligned}&\Big\{(\theta_3^{''})_{xx} + \big[(\theta_3^{''})_x + B_3\Phi_y\big]\cdot\big[(\theta_3^{''})_x - B_3\Phi_y\big] + (\theta_3^{''})_{yy} + \\ &+ \big[(\theta_3^{''})_y + B_3\Phi_x\big]\cdot\big[(\theta_3^{''})_y - B_3\Phi_x\big]\Big\}\cdot(\sin B_3\Phi + \\ &+ \cos B_3\Phi) + \big[2\cdot(\theta_3^{''})_x \cdot B_3\Phi_x + B_3\Phi_{xx} + 2\cdot(\theta_3^{''})_y \cdot \\ &\cdot B_3\Phi_y + B_3\Phi_{yy}\big]\cdot(\cos B_3\Phi - \sin B_3\Phi) = 0\end{aligned} \tag{12}$$

In the case of equality to zero, the brackets, $\big[(\theta_3^{''})_x + B_3\Phi_y\big]$, $\big[(\theta_3^{''})_y - B_3\Phi_x\big]$ in equation (12) establish a relation in the form of:

$$(\theta_3^{''})_{xx} = -B_3\Phi_{yx},\ (\theta_3^{''})_{yy} = -B_3\Phi_{xy}$$

$$B_3\Phi_{xx} = (\theta_3^{''})_{yx},\ B_3\Phi_{yy} = (\theta_{xy}^{''})$$

The last correlations correspond to Cauchy-Rieman condition and are functions determined by the Laplace equation corresponding to relation (11).

From the comparison of the solutions (7) to (11) (conditions superimposed on functions) it was concluded that $\theta_3^{''} = -B_3\theta$ for the stressed and deformed conditions and the temperature fields can be used to determine a common parametric function, which is included into the fields of the stresses, the deformations, the rates of deformation and the temperatures, allowing us to express them mathematically, one with another. Thus,

$$\exp(-\theta) = \left(\frac{H_i}{2\cdot C_\xi}\right)^{\frac{1}{B_1}} = \left(\frac{\Gamma_i}{2\cdot C_\varepsilon}\right)^{\frac{1}{B_2}} = $$

$$= \left(\frac{T}{C_T\cdot(\sin B_3\Phi + \cos B_3\Phi)}\right)^{\frac{1}{B_3}}$$

With substitution into an expression for resistance to deformation, we obtain

$$T_i = \chi\big(H_i\big)^{\frac{A_1'}{B_1}}\cdot\big(\Gamma_i\big)^{\frac{A_2'}{B_2}}\cdot(T')^{\frac{A_3'}{B_3}} \tag{13}$$

The form of expression (13) corresponds to a dependence of the yield stress from the rate, the degree of deformation and the temperature proposed in[1].

3 ANALYSIS OF THE RESULTS

In the analysis the expressions (6) are used to study the stressed condition of the plastic medium in the case of a flat upsetting of rough plates. If the problem is reduced to a more simple mathematical model ($A_2' = A_3' = 0$), expression (6) will correspond to the solutions in[5].

$$\begin{aligned}k &= C_\sigma \cdot \exp(-A_1'\theta)\\ \tau_{xy} &= C_\sigma \cdot \exp(-A_1'\theta)\cdot\sin(A\Phi)\\ \sigma_x &= C_\sigma \cdot \exp(-A_1'\theta)\cdot\cos(A\Phi) + \sigma_0 + f(y) + C\\ \sigma_y &= -C_\sigma \cdot \exp(-A_1'\theta)\cdot\cos(A\Phi) + \sigma_0 + f(x) + C\end{aligned} \tag{14}$$

Applying the condition for plasticity $\sigma_0 = -2k\cdot\cos A\Phi$, $C = k_0$, the functions $A\Phi$ and θ become harmonic. Starting from the Laplace equation and the Cauchy-Rieman conditions we obtain the expressions for determining the functions in the form of the coordinate polynomial.

$$A\Phi = AA_6\cdot x\cdot y - AA_{13}\cdot x\cdot y\cdot(x^2 - y^2)$$

$$\theta' = -\Big\{0.5\cdot AA_6\cdot(x^2 - y^2) - AA_{13}\cdot\big[0.25\cdot(x^4 + y^4) - 1.5\cdot x^2\cdot y^2\big]\Big\}$$

The constants in the expressions were determined as proceeding from the real boundary conditions

$$AA_6 = 4\cdot\frac{\Psi_0}{l\cdot h},\ AA_{13} = 16\cdot\Psi_1\cdot\frac{l - 2\cdot h}{l^3\cdot h\cdot(l+h)}$$

$$\Psi_0 = \mathrm{arctg}\,\big[2\cdot f\cdot(1-f)\big]$$

$$\Psi_1 = \mathrm{arctg}\,\big[1.7\cdot f\cdot(1-f)\big]$$

and the coefficient

$$C_\sigma = \frac{k_0}{\cos A\Phi_0}\cdot\exp(-\theta_0')$$

$$A\Phi_0 = AA_6\cdot\frac{l\cdot h}{4} - AA_{13}\cdot\frac{l\cdot h}{4}\cdot\left(\frac{l^2}{4} - \frac{h^2}{4}\right)$$

$$\begin{aligned}\theta_0' = -A\theta_0 = -\Bigg\{&0.5\cdot AA_6\cdot\left(\frac{l^2}{4} - \frac{h^2}{4}\right) - \\ &- AA_{13}\left[0.25\cdot\left(\frac{l^4}{16} + \frac{h^4}{16}\right) - 1.5\cdot\frac{l^2\cdot h^2}{16}\right]\Bigg\}\end{aligned}$$

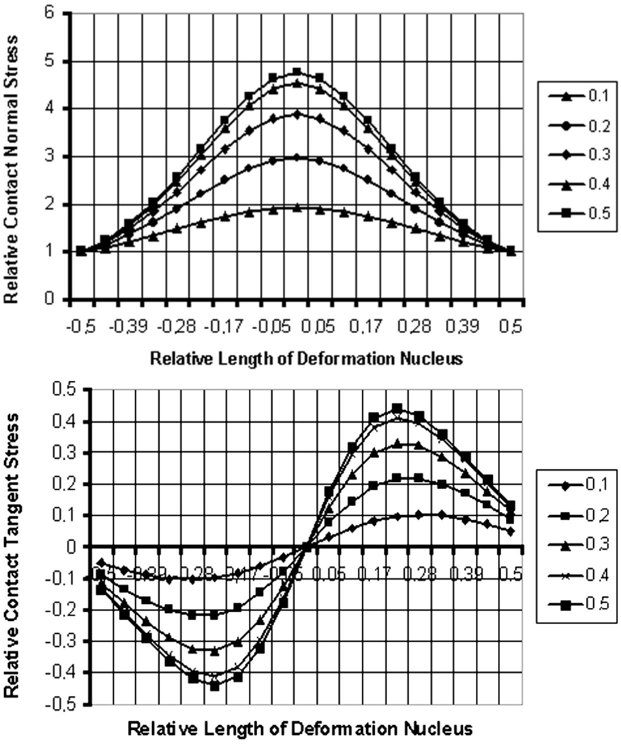

Figure 1: Distribution of the normal and tangent stresses at the contact during upsetting with rough strikers $l/h = 8$, $f = 0{,}1...0{,}5$

Slika 1: Porazdelitev kontaktnih normalnih in tangetnih napetosti pri krčenju s težkimi kladivi $l/h = 8$, $f = 0{,}1...0{,}5$

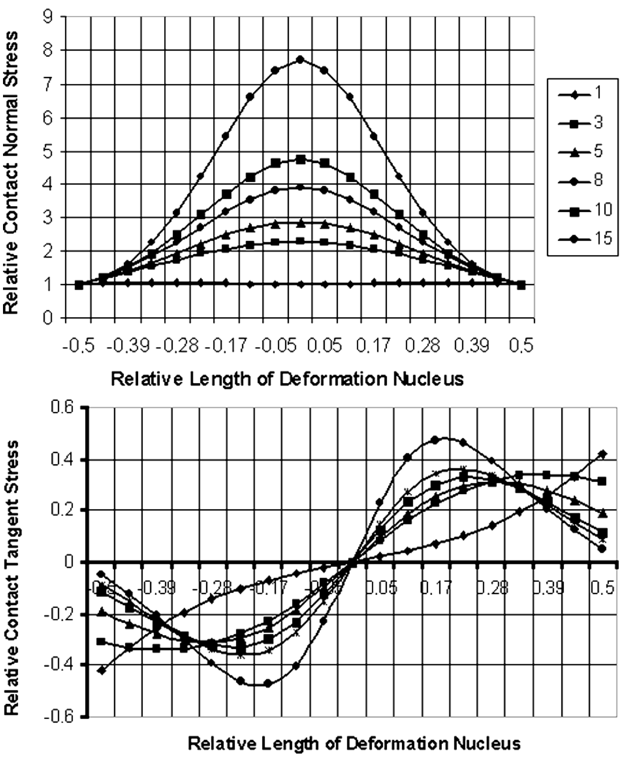

Figure 2: Distribution of the normal and tangent stresses at the contact during upsetting with rough strikers $f = 0{,}3$, $l/h = 1...15$

Slika 2: Porazdelitev kontaktnih normalnih in tangetnih napetosti pri krčenju s težkimi kladivi $f = 0{,}3$, $l/h = 1...15$

By substituting the components of the tensor of stresses into (14), we obtain

$$\sigma_x = -k_0 \cdot \frac{\exp(\theta' - \theta_0')}{\cos A\Phi_0} \cdot \cos A\Phi + k_0$$

$$\sigma_y = -3k_0 \cdot \frac{\exp(\theta' - \theta_0')}{\cos A\Phi_0} \cdot \cos A\Phi + k_0$$

$$\tau_{xy} = k_0 \cdot \frac{\exp(\theta' - \theta_0')}{\cos A\Phi_0} \cdot \sin A\Phi \qquad (15)$$

The results of the calculation according to equations (15) in **Figures 1 to 4** show that the distribution of the contact stresses is related to the factor of the shape of the stress nucleus l/h and the friction coefficient f, the relative normal stresses $\sigma_y/2k_0$ and the relative tangent stresses τ_{xy}/k_0. The results of the calculation correspond to the real distribution diagrams of the contact stresses.[6] It should be emphasized that expressions (15) are uniform for the entire nucleus of deformation and there is no need to break it into separate zones of contact friction.[7] **Figures 5 and 6** show the distribution of stresses

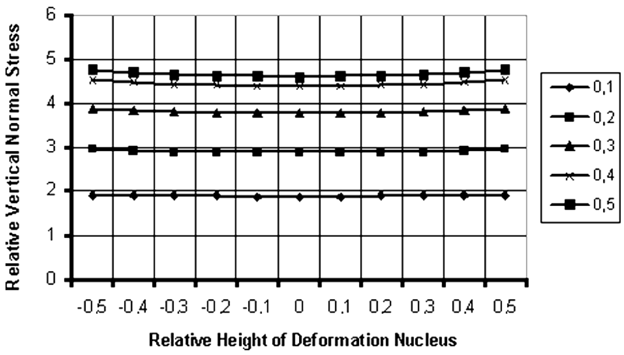

Figure 3: Distribution of the normal stresses along the plate height during the upsetting with rough strikers $l/h = 8$, $f = 0{,}1...0{,}5$

Slika 3: Porazdelitev normalnih napetosti po višini plošče pri krčenju s težkimi kladivi: $l/h = 8$, $f = 0.1…0.5$

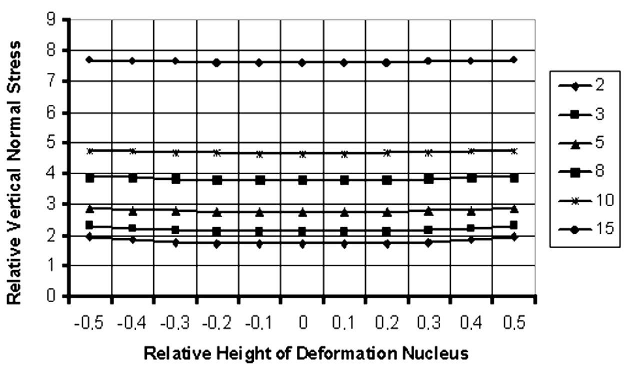

Figure 4: Distribution of the normal stresses along the strip height during the upsetting with rough strikers $f = 0{,}3$, $l/h = 2...15$

Slika 4: Porazdelitev normalnih napetosti po višini plošče pri krčenju s težkimi kladivi $f = 0{,}3$, $l/h = 2...15$

in the nucleus of deformation, which also include the shape factor and the friction coefficient.

The obtained results display qualitatively and quantitatively the general patterns of the distribution fields of the tensor of stresses over the entire nucleus of deformation. The results meet fully the requirements of the boundary conditions. In particular, the proposed procedure and expressions[15] can be recommended for the calculation of various problems in applications.

The proposed complex model of a plastic medium based on the closed solution can be considered as a generalization of the theory of plasticity, uniting the theories of deformation and of plastic yielding.

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LIST OF SYMBOLS

σ – normal components of the stress tensor;
τ – tangential components of the stress tensor;
ξ – linear components of the strain-rate tensor;
γ – shear components of the strain-rate tensor;
ε – linear deformation along the axes x and y;
χ – factor of correspondence between the tangent stress and the temperature-deformation parameter of the centre to deformation;
$\Gamma_{\rm i}$ – intensity of the shift of the deformation;
$\tau_{\rm n}$ – tangential contact stress on an arbitrary inclined area;
α – angle of inclination of the contact area;
k – shearing plastic deformation strength;
Φ – harmonic function depending on the coordinates of the deformation zone and the argument of a trigonometric function;
Φ_0 – argument of trigonometric function for $x = \pm l/2$ and $y = \pm h/2$;
A – constant characterizing the trigonometric function for the state of stress of the plastic medium;
A_6, A_{13} – constant factor, characterizing the shearing tangent stress in the zone of reduction
Ψ_0, Ψ_1 – values taking into account the influence of the factor of friction;
B – constant value characterizing the trigonometric function for the state of strain of the plastic medium;
θ_0 – factor exhibitors for $x = \pm l/2$ and $y = \pm h/2$;
$\theta^{\cdot}$ – harmonic function, exponential index, characterizing the shearing stress distribution in the zone of reduction;
$\theta^{\cdot\cdot}$ – a harmonic function, exponential index, characterizing the distribution of the rate of shearing in the zone of reduction;
C_σ – constant value determining the state of stress of the plastic medium;
C_ξ – constant value characterizing the state of strain of the plastic medium;
T – temperature in the k-th point;
$T_{\rm i}$ – intensity of tangential stress;
$H_{\rm i}$ – intensity of shearing rates;
C_T – constant value characterizing the temperature field;
a – coefficient of temperature conductivity;
l and h – length and height of the deformation nucleus during strip upsetting;
f – friction coefficient;
k_0 – contact shear resistance at the beginning of the deformation nucleus