

# APPLICATION OF THE THEORY OF PHYSICAL SIMILARITY FOR THE FILTRATION OF METALLIC MELTS

## UPORABA TEORIJE FIZIKALNE PODOBNOSTI ZA OPIS FILTRIRANJA KOVINSKE TALINE

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The Bernoulli equation is the basis for the primary description of the flow of a real metallic melt through the pouring system for the filling of the casting mould with an inserted ceramic filter. In principle, a modified, dimensionless form of the Bernoulli equation can be used for the determination of the loss coefficient  $\xi$  as a general function of the dimensionless criteria – the Reynolds, Froude and Euler numbers. It was verified by modelling the flow of the modelling liquid (in this case water) through ceramic filters. In the same interval of Reynolds numbers the loss coefficient  $\xi$  was greater for foam filters than for filters with direct holes (strainers); however, the outlet coefficient  $\mu$  of the foam filters was, in identical conditions, significantly lower than that of filters with direct holes.

Key-words: similarity criteria, modelling, steel flow, ceramic filters, steel filtration

Bernoullijeva enačba je podlaga za opis pretoka taline skozi livni sistem z vstavljenim keramičnim filtrom. Načeloma je mogoče uporabiti modificirano brezdimenzijsko obliko Bernoullijeve enačbe za določitev koeficienta izgube  $\xi$  kot funkcije brezdimenzijskega kriterija – Reynoldsovega, Froudovega ali Eulerjevega števila. S pretokom modelne tekočine (vode) je bilo preverjeno, da je pri enakem intervalu Reynoldsovih števil koeficient izgube  $\xi$  večji pri penastem filtru kot pri filtru z direktnimi luknjicami, vendar pa je izhodni koeficient  $\mu$  za penaste filtre pri enakih pogojih pomembno nižji kot pri filtrih z neposrednimi luknjicami.

Ključne besede: merila podobnosti, modeliranje, tok jeklene taline, keramični filtri, filtriranje jeklene taline

## 1 INTRODUCTION

The theory of physical similarity makes it possible to investigate the regularities of physical and other phenomena with similar behaviour, and it is possible to conclude from the known behaviour of one phenomenon, the behaviour of a second. This theory is based on similarity criteria, i. e., on dimensionless quantities (similarity numbers), which can substitute in the investigated phenomenon for dimensional physical quantities. The determination of similarity criteria is the basic task for the application of the theory of physical similarity of an investigated problem, since, on the one hand, they *reduce the number of variables* describing the given problem, and, on the other hand, they *specify the similarity relations* between the pertinent phenomenon and its model. The theory of physical similarity is used for the determination of similarity criteria and afterwards, for criteria dependencies, three methods of generalised variables, including the dimensional analysis, the analysis of the mathematical model and finally the analysis of the physical model.

The objective of this study is to prove that the technology of filtration of metallic melts can be efficiently

investigated by the application of the theory of physical similarities using appropriately determined dimensionless similarity numbers – i. e., criteria. This study is based on extensive previous and current experimental research <sup>1-6</sup>.

## 2 PRINCIPAL DIMENSIONLESS CRITERIA OF SIMILARITY FOR THE FILTRATION OF METALLIC MELTS

The *Bernoulli* equation is the basis for the primary description of the flow of a real metallic melt through the pouring system for the filling of the relevant casting mould with an inserted ceramic filter. With respect to the pressure losses caused during the flow through the filter, this equation can have the following form:

$$\frac{\rho w^2}{2} + h\rho g + p + \xi \frac{wv\rho}{d} = \text{const.} \quad (1)$$

where  $\rho w^2/2$  (Pa) represents the kinetic energy,  $h\rho g$  (Pa) represents the positional or gravitational energy of the capacity of a liquid unity, which is determined by gravity, and  $p$  (Pa) represents the potential pressure energy, which is usually dependent on the external

forces being exerted. The values in the equation are:  $w$  = the real liquid flow-rate ( $\text{m s}^{-1}$ ),  $\rho$  = the density of the flowing liquid ( $\text{kg m}^{-3}$ ),  $h$  = the real position, the real height of the flowing liquid (m),  $g$  = the acceleration due to gravity ( $\text{m s}^{-2}$ ),  $\xi$  = a dimensionless loss coefficient expressing the filter's hydraulic resistance (-),  $\nu$  = the kinematic viscosity of the flowing liquid ( $\text{m}^2 \text{s}^{-1}$ ) and  $d$  = the diameter of the filter capillaries with direct holes (m). The pressure of the flowing real liquid  $p$  is expressed in a basic unit (Pa) with the basic physical dimension of ( $\text{kg s}^{-2} \text{m}^{-1}$ ). It follows from the *Bernoulli* equation that the sum of the kinetic, positional, pressure and loss energy of an ideally flowing liquid remains constant, i.e., *const* in volume unit at each place during its flow.

The original *Bernoulli* equation (1) is transformed by division with the expression  $\rho w^2/2$  (Pa) to a dimensionless form

$$1 + \frac{2hg}{w^2} + \frac{2p}{\rho w^2} + \xi \frac{2\nu}{wd} = \frac{2 \text{const.}}{\rho w^2} \quad (2)$$

which contains the already standard (internationally) adopted dimensionless similarity criteria, namely: the Froude criterion  $Fr = w^2(hg)^{-1}$ , the Euler criterion  $Eu = p\rho^{-1} w^{-2}$  and the Reynolds criterion  $Re = wd\nu^{-1}$ . With the use of these similarity criteria (numbers) it is possible to express the *Bernoulli* equation in a dimensionless form:

$$1 + \frac{2}{Fr} + 2Eu + \frac{2\xi}{Re} = \frac{2 \text{const.}}{\rho w^2} \quad (3)$$

The Froude criterion  $Fr = w^2(hg)^{-1}$  expresses the ratio of the inertia and the gravitational forces in the flowing metallic liquid, including its undulation, vortices and also surface phenomena. It is often also called the Froude criterion 1,  $Fr_1$ , in contrast to the Froude criterion 2,  $Fr_2$ , expressed in the form  $Fr_2 = w(hg)^{-1/2}$ , which characterises the so-called "kinetic head".

The Euler criterion  $Eu = p(\rho w^2)^{-1}$  expresses the ratio of the pressure and the inertia forces. It characterises the loss of pressure during the flowing of a real metallic liquid and influences the hydraulic resistance of the flowing liquid caused by viscous forces (viscosity).

The Reynolds criterion  $Re = wd\nu^{-1}$  belongs to the basic hydrodynamic criteria and it characterises the ratio of the inertia and the viscous forces, i.e., the forces caused by the viscosity of the filtered metallic melt. Its absolute value contains information about the basic character of the flowing of viscous liquids, laminar or turbulent, and also the information about the transition from one type of form of flowing to another.

The modified dimensionless form of the *Bernoulli* equation (3) can, in principle, be used for the determination of the loss coefficient  $\xi$ , as a general function of dimensionless criteria

$$\xi = \frac{Re}{2} \left\{ \frac{2 \text{const.}}{\rho w^2} - \left( 1 + \frac{2}{Fr} + 2Eu \right) \right\} \quad (4)$$

which indicates that during the flowing of a metallic melt through the ceramic filter the loss coefficient is proportional to the Reynolds number, whereas the constant of proportionality is also dependent on the Froude and Euler numbers and on other parameters of the Bernoulli equation. The expression  $2 \text{const.} (\rho w^2)^{-1}$  is also dimensionless, since the original physical dimension of this constant is in pascal (Pa).

### 3 MODELLING OF A METALLIC MELT FLOW THROUGH CERAMIC FILTERS

The practical use of the interdependence between the loss coefficient and the used ceramic filter requires that an equation is determined that contains certain specific numeric values. It is possible to determine such an equation by an appropriately chosen and arranged type of modelling. For these purposes the hydraulic characteristics of filters with direct holes (strainers) and foam filters were chosen for the modelling. Water served as the modelling liquid, since its viscosity is quantitatively comparable with the viscosity of liquid steel. The measurement of the hydraulic parameters during the flowing of a liquid through ceramic filters was carried out on the water measuring line in the laboratory of the Department of Airplanes and Engines at the Military Academy in Brno.

The results of the measurements for two different types of filters are described in detail in the reports<sup>7-9</sup>:

a) For the same type of filter with direct holes (strainer) manufactured by the company Keramtech Žacléř, s. r. o., Czech Republic, a filter number of 0217, a diameter of 70 mm and a basic filter height (thickness)  $L = 10$  mm with 19 holes of diameter  $D = 6$  mm, in which a total of six filter-slenderness ratios were measured. The slenderness ratios were defined as the ratio of the filter height (thickness)  $L$  and the diameter of one filter-capillary aperture  $D$ . The changes of slenderness were achieved by changes of the height (thickness) of the same type of filters and by their serial ordering (stacking).

b) For the same type of foam filter (*schaum filter*) with the dimensions (90×90×25) mm (8.0 ppi), manufactured by the company FOSECO, in which two different hole slendernesses were measured.

The first series of modelling, the results of which are briefly presented in this study, was focused on the determination of the loss coefficient  $\xi$  and of the filter outlet coefficient  $\mu$ . The specific energy loss  $e_z$  was defined for the purposes of the assessment of selected filters by the relation  $e_z = (\xi w^2/2)$  ( $\text{m}^2 \text{s}^{-1}$ ), which means that the energy loss is directly proportional to the filter loss coefficient (filter specific resistance) and the square of the flow rate of the modelling liquid – the water in the pipeline of the modelling system. The Reynolds number

of the investigated filters was defined by the standard relation

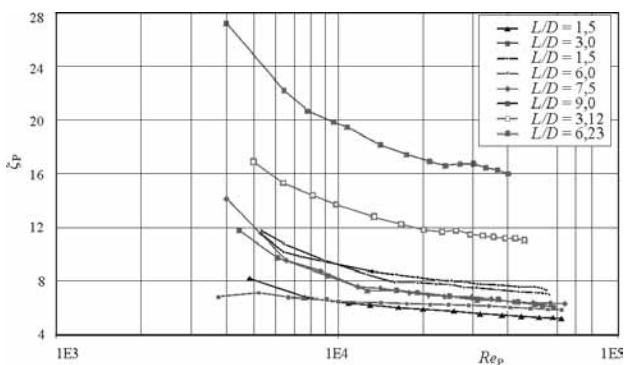
$$Re_F = \frac{w_F D_{hF}}{\nu} \quad (5)$$

where  $w_F$  is the flow rate of the liquid in the filter and  $D_{hF}$  is the hydraulic diameter of the filter, which is equal to the diameter of one of its apertures (capillaries). The filter outlet coefficient  $\mu$  was defined as the ratio of the real volume flow of the modelling liquid (water) with respect to the friction losses and the possible contraction of the water jet to the ideal volume flow through the filter, determined from the equations for the flowing of an ideal liquid<sup>7</sup>.

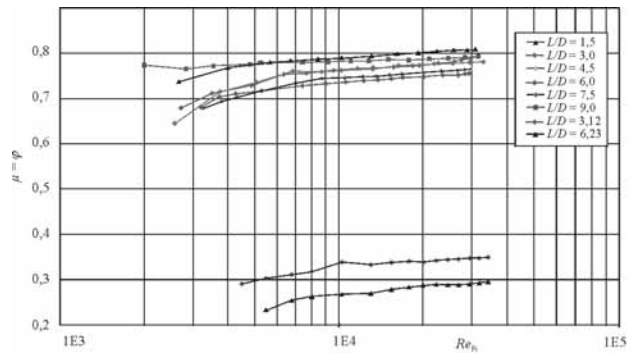
#### 4 RESULTS OF THE MODELLING OF THE FLOW THROUGH CERAMIC FILTERS

In **Figure 1** the dependence of the *loss coefficient*  $\xi_p$  of both types of ceramic filters, as determined by modelling, on the Reynolds number  $Re_F$  and the slenderness of filter apertures  $L/D$  is shown. The loss coefficient is considered as the *local resistance of the filter* inserted into the pipeline (index P), and as the resistance corresponding to the state of the modelling liquid (i.e., water at a temperature of 20 °C) in the filter (index F). The two curves in the upper part of the diagram are valid for foam filters, while the group of curves in the bottom part of diagram relates to filters with direct holes (strainers). With increasing Reynolds number and decreasing filter slenderness the coefficient  $\xi_p$ , expressing the filter's resistance and the energy loss of the flowing liquid, decreases. However, for the foam filters the decrease of the coefficient  $\xi_p$  with the decrease of the filter's slenderness  $L/D$  is distinctly larger than for filters with direct holes.

The modelling of the *filter-outlet coefficient*  $\mu$  is shown in **Figure 2**. The two curves in the bottom part of the diagram are related to foam filters, while the group of curves in the upper part of the diagram are related to filters with direct holes. The outlet coefficient  $\mu$  also



**Figure 1:** Dependence of the coefficient of the local resistance of the ceramic filters on the Reynolds number and the filter's slenderness  
**Slika 1:** Odvisnost koeficienta lokalnega upora keramičnega filtra od Reynoldsovega števila in od vitkosti filtra



**Figure 2:** Dependence of the filter-outlet coefficient on the Reynolds number and the ceramic filters' slenderness  
**Slika 2:** Odvisnost izhodnega koeficienta od Reynoldsovega števila in od vitkosti filtra

expresses the contraction  $\varphi$  of the modelling liquid flowing out of the filter. It is obvious that the foam filters have, in comparison with the filters with direct holes, significantly lower values of the outlet coefficient  $\mu$ , although the slendernesses of the apertures (capillaries) were otherwise identical. The values of the outlet coefficients  $\mu$  for the foam filters vary in the interval 0.23 to 0.35, while the values of the outlet coefficients  $\mu$  for the filters with direct holes are within the interval 0.65 to 0.80, and the Reynolds numbers are very close  $Re_{Fi} = 2 \cdot 10^3$  to  $3.5 \cdot 10^4$  (**Figure 2**). It is worth noting that the Reynolds number  $Re_{Fi}$  in the diagram shown in **Figure 2** is linked to the Reynolds number  $Re_p$  in the diagram shown in **Figure 1** by equation (6) in the following manner:

$$Re_p = \frac{Re_{Fi}}{\sqrt{1 + \xi_p}} \quad (6)$$

During the modelling measurements the theory of physical similarities was strictly respected and the obtained results were expressed with dimensionless similarity numbers.

#### 5 CONCLUSIONS

The results of the modelling measurements shown in **Figures 1** and **2** indicate that it is characteristic for both different types of ceramic filters (i.e., filters with direct holes and foam filters) that lower the values of the coefficient  $\xi_p$ , expressing the filter losses or the resistance to the flowing of the liquid through the filter, correspond to higher values of the coefficient  $\mu = \varphi$ , expressing the contraction of the liquid flow flowing out from the filter aperture (capillary) and vice versa. The higher hydraulic resistance during the flow of the steel melt through the foam filters favourably affects the micro-mechanism of filtration. The hydraulic resistance influences the filtration of metallic liquid through ceramic filters with direct holes (capillaries) to a considerably lesser degree. **Table 1** summarises the values for

**Table 1:** Comparison of the kinematic viscosity of liquids flowing through the filters – water and steel**Tabela 1:** Primerjava kinematične viskoznosti tekočin, ki se pretakajo skozi filter – voda in jeklena talina

Kinematic viscosity $\nu / \text{m}^2 \text{s}^{-1}$	Model – water <sup>10</sup>		Reality – carbon steel <sup>11</sup>	
	Temperature /°C	Viscosity / $\text{m}^2 \text{s}^{-1}$	Temperature /°C	Viscosity / $\text{m}^2 \text{s}^{-1}$
	20	$1.55 \cdot 10^{-7}$	1600	$6.23 \cdot 10^{-7}$

the kinematic viscosity of the water at the temperature of the modelling of flow through the filter and carbon steel containing the mass fraction of C 0.25 % at the casting temperature.

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