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ASYMMETRICAL NOTCHES IN PLATES

NESIMETRIČNE ZAREZE V PLOŠČAH

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Stress concentration due to notches in machine parts is nowadays well known, and design codes provide data to take into account this phenomenon. Several shapes of notches are referenced and, in the case of plates, one-side- or two-sides-notched plates are considered. In the latter case, only the symmetrical case (same shape) is presented. This work analyses the case of non-symmetrical plates loaded in tension, i.e., that have notches of different shape on opposite sides. Unexpected results relating to the stress-concentration factor have been obtained and have been analysed using the global stress distribution.

Key words: stress concentration factor, asymmetrical notches, FEM based calculations and simulations

V inženirski praksi se pogosto med konstruiranjem in oblikovanjem srečamo z različnimi oblikami zarez. Dobro je poznano, da so zareze izvir koncentracije napetosti. Velikost koncentracije napetosti izrazimo s faktorjem koncentracije napetosti.

Faktor koncentracije elastičnih napetosti je definiran kot razmerje med maksimalno in globalno (bruto) napetostjo (bruto faktor koncentracije napetosti). Lahko pa ga definiramo tudi kot neto faktor koncentracije napetosti. V referenčni literaturi so ti faktorji koncentracije napetosti zbrani tabelarično ali grafično za različne značilne geometrije oziroma oblike zarez. V glavnem najdemo v literaturi podatke le za simetrične zareze, kot so na primer polkrožne, U- in V-zareze.

V literaturi skoraj nikjer ne najdemo podatkov za različne vrste nesimetričnih zarez. Zato moramo uporabiti računalniške simulacije, ki nam omogočajo izračun faktorjev koncentracije napetosti nesimetričnih problemov. V prispevku je predstavljenih nekaj primerov računalniških izračunov nesimetričnih zarez in primerjav s simetričnimi zarezami. V nekaj primerih smo dobili nepričakovane rezultate, ki smo jih analizirali z uporabo globalne porazdelitve napetosti.

Ključne besede: elastične napetosti, asimetrične zareze, metode končnih elementov

1 INTRODUCTION

Notches are frequently encountered by engineers when they are designing a mechanical part. These notches are necessary for joining systems, for clamping shafts, etc. Notches are important in design problems because they induce stress concentration. There are two effects: the first is the reduction of the global section related with the depth of the notch, the second is the stress gradient related to the shape of the notch. In this work we are particularly interested in the case of two opposite notches in plates subjected to uniform tension and the stress-concentration effect in relation to the shape of the notches.

In the case of two notches they are generally identical, and the problem is symmetrical. Of course, in the case of a shaft, the problem is axisymmetrical. However, in the case of plates, non-symmetrical problems can be encountered. This means that both notches are not identical, e. g., a semi-circular and a U-shaped notch, or a U-shaped and a V-shaped notch, etc.

Solutions for symmetrical problems are easily found in the literature. However, non-symmetrical problems are not referenced and it is necessary to make a computation for these cases. This is the object of the work here presented.

2 COMPUTATIONS

The problems under investigation are the cases of non-symmetrical notches in plates subjected to tension. Symmetrical cases will be computed first, as a basis for a comparison and for the verification of the precision of the computation with respect to the data found in the literature[1]. Then, using similar numerical models, non-symmetrical cases will be computed and compared to the results obtained for symmetrical cases. A typical geometry is shown in **Figure 1**. The cases computed and presented here are summarized in **Table 1**.

Table 1: Computed geometrical combinations
Tabela 1: Izračunavane geometrične kombinacije

	Semi-circular (SC)	U-shaped (U)	V-shaped (V)
Semi-circular (SC)	SC – SC	SC – U	SC – V
U-shaped (U)		U – U	U – V
V-shaped (V)			V – V

Throughout this paper, the total width of the plate is referred to as D, the ligament (the remaining section between the two notch tips) is d, and the radius of the notch tip is r. The stress-concentration factors are usually referred to as k_{tg} for the *global* stress-concentration factor, and k_{tn} for the *net* stress-concentration factor. These factors are defined as follows:

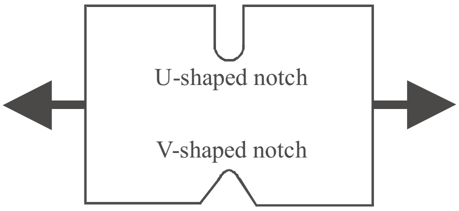

Figure 1: Typical geometry of a non-symmetrical notch problem
Slika 1: Tipična geometrija obravnavanega problema plošče z nesimetričnima zarezama

$$k_{tg} = \frac{\text{maximum stress at crack tip}}{\text{global stress}} \quad (1)$$

$$k_{tn} = \frac{\text{maximum stress at crack tip}}{\text{net stress}} \quad (2)$$

Where the *global stress* is the stress far from the notched section and the *net stress* is the stress computed in the notched section.

Moreover, all the computations are made assuming the material behaves elastically, although elasto-plastic stress-concentration factors are of major importance for design.

Finally, in the case of non-symmetrical problems, the geometry is built so that the depth of each notch is the same and the radius at the crack tip is also identical. The only varying factor is the shape of the notch. The total width of the plate is fixed at 10 mm.

3 PREPARATION OF THE COMPUTATIONS

All the computations were made with the CASTEM2000® programme, available from the French Atomic Energy Agency CEA. This programme is very suitable for research problems and is easily adaptable to our specific problem.

3.1 Meshing

As most of the configurations deal with partly circular notches, quadratic triangular elements were used to fit, as well as possible, with the shape of the specimens. For a series of specimens, the same density of mesh is preserved, so that the different computations are realized in the same conditions. A typical mesh for a U-shaped notch is given in **Figure 2**. The other meshes are similar.

One of the objectives of the mesh is to provide, at the end of the computation, the stress gradient on the ligament. For small notch radii, it is very important to have small enough elements, so that that the stress gradient is correctly taken into account. The mesh should maintain a compromise between this last condition and a reasonable number of elements, in order to reduce the time for the computation. It should be noted that all the meshes were designed automatically by the program in the same way for the large notch radii as for the small notch radii. This results in very dense meshes for a large part of the small notch radii specimens, but this should not affect the result.

3.2 Symmetry and boundary conditions

Symmetrical cases were meshed on a quarter of the plate, as the specimen exhibits two perpendicular axes of symmetry. The non-symmetrical cases, consequently, were meshed on a half specimen, as one of the axes of symmetry disappeared. The corresponding meshes are shown in **Figure 3, Figure 4** and **Figure 5**.

The boundary conditions were applied, taking into account these conditions of symmetry, so that the behaviour of the total plate is properly represented. The horizontal displacements of the nodes of the ligament were blocked up and the vertical displacement of one of these nodes was also blocked. The load was applied as an arbitrary uniform horizontal displacement of the opposite side, so that the stress can be considered as uniform far away from the ligament.

3.3 Material and behaviour

The plate material was considered as elastic and isotropic. It was taken as a standard steel with a modulus of elasticity of $2.1 \cdot 10^{11}$ Pa and a Poisson's ratio of 0.3.

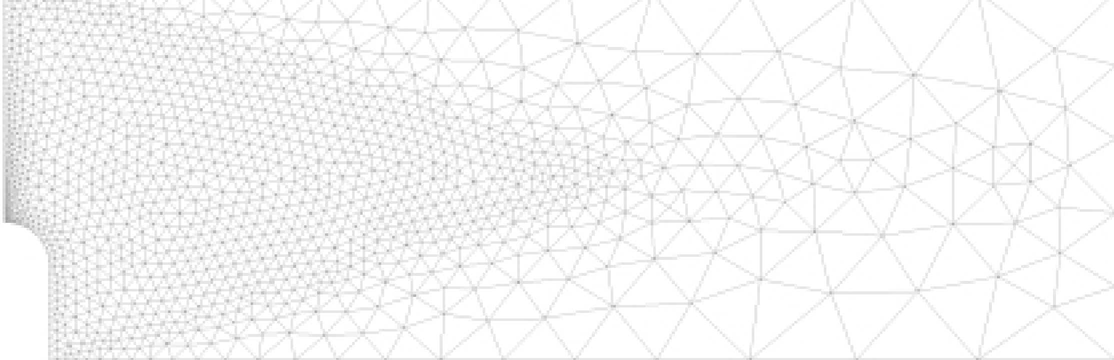

Figure 2: Typical mesh for U-shaped notches
Slika 2: Tipično zamreženje plošče z U-zarezo

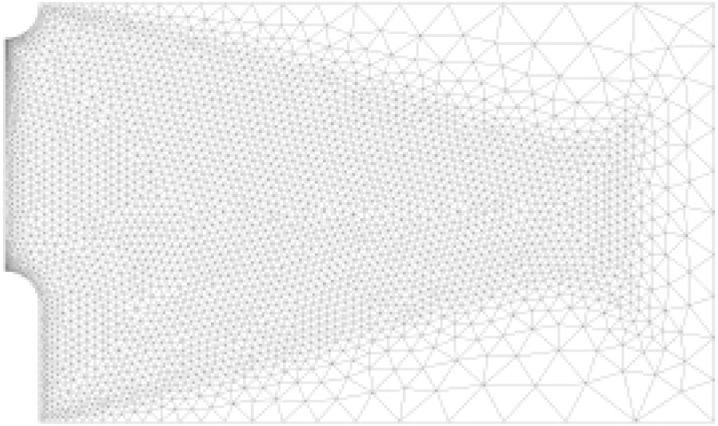

Figure 3: Typical mesh for semi-circular and U-shaped specimen
Slika 3: Tipično zamreženje plošče s polkrožno in U-zarezo

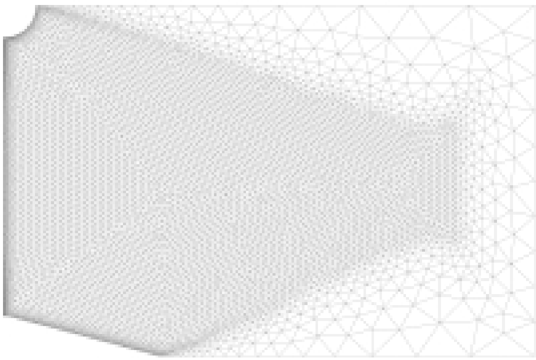

Figure 4: Typical mesh for semi-circular and V-shaped specimen
Slika 4: Tipično zamreženje plošče s polkrožno in V-zarezo

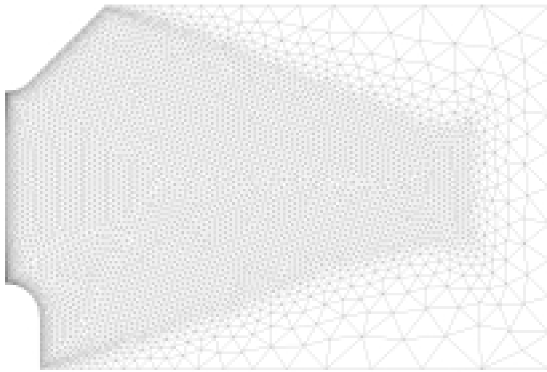

Figure 5: Typical mesh for U- and V-shaped specimen
Slika 5: Tipično zamreženje plošče z U- in V-zarezo

3.4 Computations

For a given configuration of notches, the program was launched, doing loops on the *D/d* and *r/D* ratios. The results are given in terms of the stress-concentration factor versus the ratio *r/D*, the curves being parameterised with respect to the ratio *D/d*. The computations were controlled by the visualization of the stress field in the plate. Typical examples are given in **Figure 6**, **Figure 7**, **Figure 8** and **Figure 9**.

4 RESULTS

4.1 Results on symmetrical notches

Figure 10 to **Figure 12**, show the results of k_{tn} or k_{tg} plotted versus *r/d* for different *D/d* ratios. It is easy to compare with literature[1], and an example of the literature data is given in **Figure 13**. The numerical results are very close to those found in the tables, provided the *r/d*

Figure 6: Representation of the horizontal stress distribution in a plate with a semi-circular notch
Slika 6: Predstavitev horizontalne porazdelitve napetosti v plošči s polkrožno zarezo

Figure 7: Representation of the horizontal stress distribution in a plate with semi-circular and U-shaped notches

Slika 7: Predstavitev horizontalne porazdelitve napetosti v plošči s polkrožno in U-zarezo

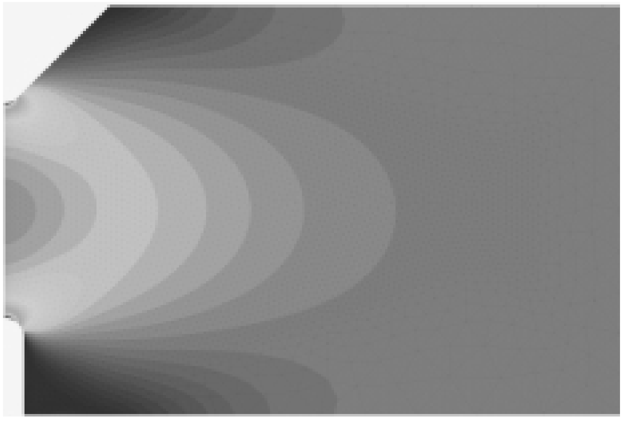

Figure 9: Representation of the horizontal stress distribution in a specimen with V-shaped and U-shaped notches

Slika 9: Predstavitev horizontalne porazdelitve napetosti v plošči z V- in U-zarezo

and *D/d* ranges are the same. This validates the FE computation.

4.2 Results for non-symmetrical notches

The results of the computations for the non-symmetrical plates are presented in **Figure 14** to **Figure 16**. The global aspect of the curves is similar to that obtained with the symmetrical cases. However, it should be noted that for the lowest values of *D/d*, in the SC-U and SC-V cases, the computed stress concentration factor decreases with *r/d*, then increases. This unexpected behaviour can be correlated with the aspect of the σ_{xx} (horizontal) stress field as shown in **Figure 7** to **Figure 9**. The cases with this behaviour are represented by **Figure 7** and **Figure 8**. It is observed that the stress field is highly non-symmetrical when, on the opposite, the stress field remains quite symmetrical in the U-V case (**Figure 9**).

To facilitate a comparison, six curves of K_{tn} versus *r/D* are plotted on the same graph for a given value of *D/d*=1.3 (**Figure 17**). It is clear that the curves in all the different cases are very close to each together, except at each end, when *r/D* is very small.

Taking into account the particularly non-symmetrical geometry of the cases under consideration, it is assumed that there is a strong bending effect that induces a stress gradient in the ligament, and that this enhances the stress concentration, and that it is more sensitive for the higher *r/D* values.

However, it should be noted that, in the SC-U case, the geometry evolves with the *D/d* and *r/d* parameters. In other words, beyond the computed threshold values of the parameters, the U-shaped notch becomes a partially circular notch, and then it vanishes. This geometrical effect is shown in **Figure 18**, with vertical lines showing the limit between the U-shaped notches and the partially

Figure 8: Representation of the horizontal stress distribution in a plate with semi-circular and V-shaped notches

Slika 8: Predstavitev horizontalne porazdelitve napetosti v plošči s polkrožno in V-zarezo

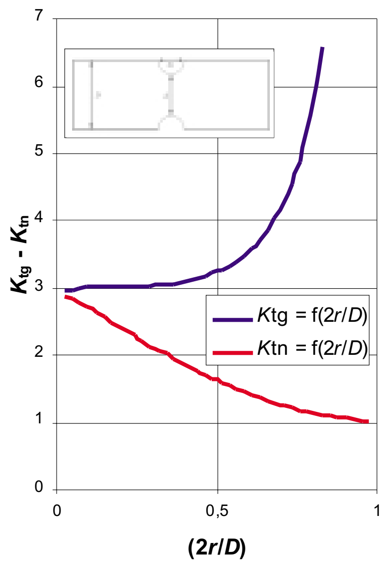

Figure 10: Stress-concentration factors for a plate in tension with two semi-circular notches

Slika 10: Faktor koncentracije napetosti za natezno obremenjeno ploščo z dvema polkrožnima zarezama

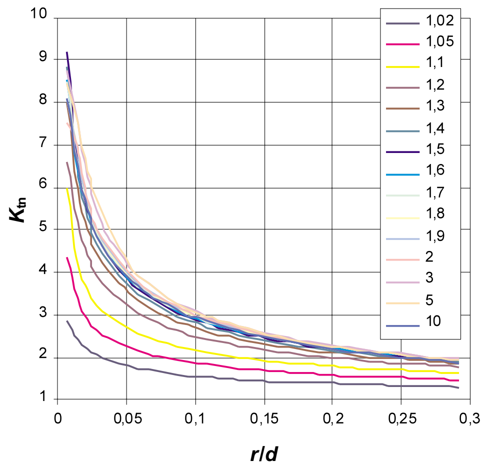

Figure 11: Stress-concentration factor for a plate in tension with two U-shaped notches

Slika 11: Faktor koncentracije napetosti za natezno obremenjeno ploščo z dvema U-zarezama

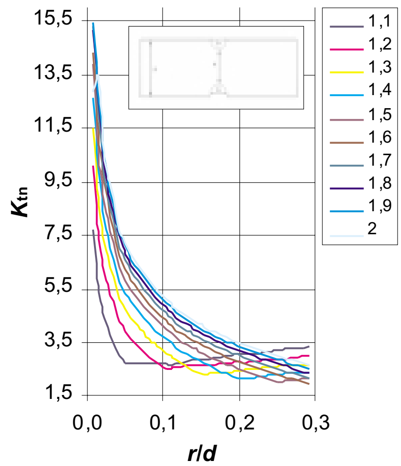

Figure 14: Stress-concentration factors for a plate with semi-circular and U-shaped notches

Slika 14: Faktor koncentracije napetosti za natezno obremenjeno ploščo s polkrožno in U-zarezo

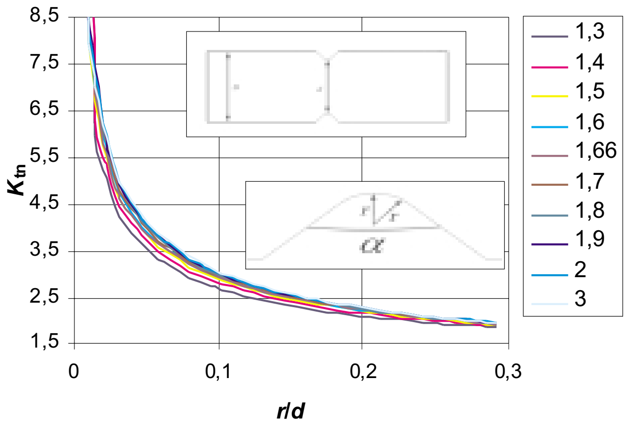

Figure 12: Stress-concentration factor for a plate in tension with two V-shaped notches (α = 60°)

Slika 12: Faktor koncentracije napetosti za natezno obremenjeno ploščo z dvema V-zarezama (α = 60°)

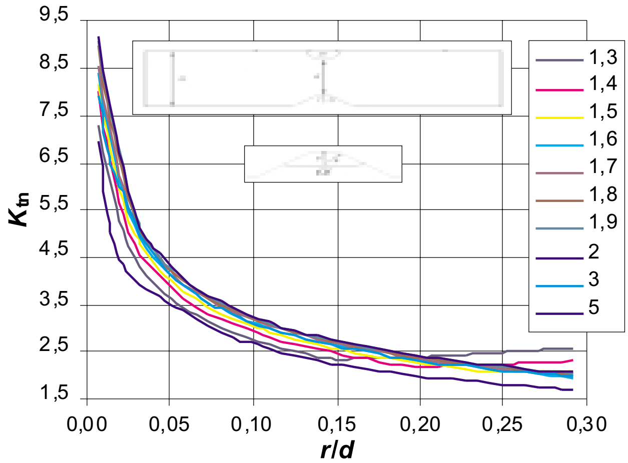

Figure 15: Stress-concentration factors for a plate with semi-cricular and V-shaped notches

Slika 15: Faktor koncentracije napetosti za natezno obremenjeno ploščo s polkrožno in V-zarezo

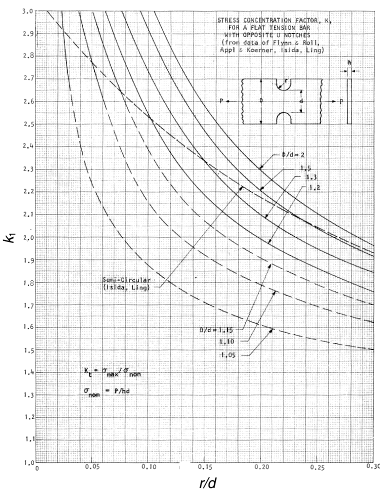

Figure 13: The stress-concentration factor for a plate with two U-shaped notches following the literature[1]

Slika 13: Faktor koncentracije napetosti za natezno obremenjeno ploščo z dvema U-zarezama[1]

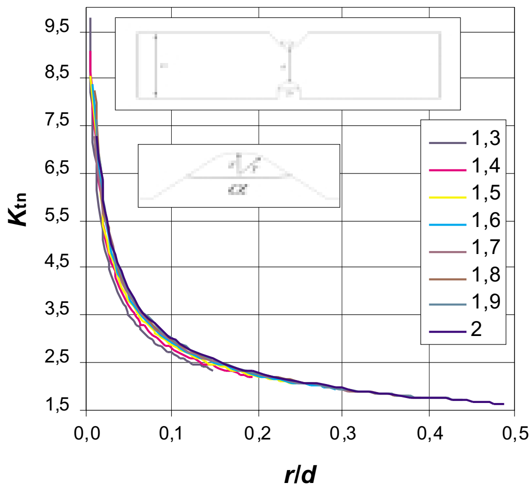

Figure 16: Stress-concentration factor for a plate with U- and V-shaped notches

Slika 16: Faktor koncentracije napetosti za ploščo z U- in V-zarezo

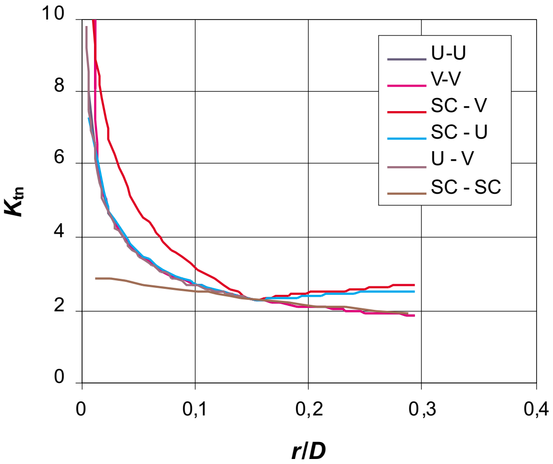

Figure 17: Comparison between stress-concentration factors for the different cases and D/d = 1.3 and α = 60° (V-shaped notches)

Slika 17: Primerjava med faktorji koncentracije napetosti za različne geometrijske primere in D/d = 1,3 in α = 60° (V-zareza)

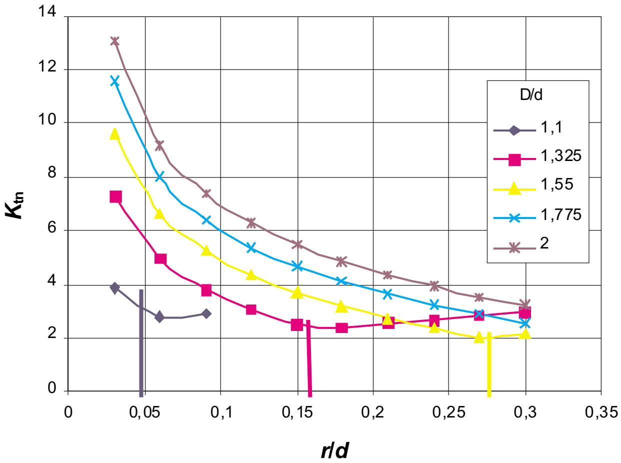

Figure 18: Geometrical limits for computation in the SC-U case. Vertical lines indicate transition from U shape to partially circular shape. The curves are limited to the point where the U-notch vanishes.

Slika 18: Geometrične meje za izračun primera plošče s polkrožno in U-zarezo. Vertikalne linije označujejo prehod iz U- v delno polkrožno obliko. Krivulje so omejene do točke, kjer U-zareza izgine.

circular notches, and the curves are limited to the point where the notch vanishes.

Of course, the conclusions that have been derived for the SC-U case are also valid for the SC-V case.

5 CONCLUSIONS

A numerical determination of the elastic stress-concentration factors in plates subjected to tension with non-symmetrical notches has shown that the computed values of K_{tn} are similar to that found in the literature for symmetrical cases. However, noticeable differences are reported for extreme values of the *r/D* ratio. Moreover, for the lowest values of *D/d*, it is observed for SC-U and SC-V geometries that the stress-concentration factor increases for $r/D > 0.15$. This unexpected behaviour is attributed to the large asymmetry of the specimen.

To confirm these computations, it will be necessary in the future to realise experimental measurements of the stress-concentration factor. Moreover, it would be interesting to investigate such geometries for a calculation of the elastoplastic stress- and strain-concentration factors.

6 REFERENCES

[1] R.E. Peterson, Stress and concentration factors, 2nd ed., Wiley, New York, 1974